

# Domain walls in three dimensional gauged supergravity

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**ABSTRACT:** We explicitly construct two Chern-Simons gauged supergravities in three dimensions with  $N = 4$  and  $N = 8$  supersymmetries and non-semisimple gauge groups. The  $N = 4$  theory has scalar manifold  $SO(4, 3)/SO(4) \times SO(3)$  with the gauge group  $SO(3) \ltimes (\mathbf{T}^3, \hat{\mathbf{T}}^3)$ . The theory describes  $(1, 0)$  six dimensional supergravity reduced on an  $SU(2)$  group manifold. The equivalent Yang-Mills type gauged supergravity has  $SO(3)$  gauge group coupled to three massive vector fields. The  $N = 8$  theory is described by  $SO(8, 8)/SO(8) \times SO(8)$  scalar manifold, and the gauge group is given by  $SO(8) \ltimes \mathbf{T}^{28}$ . The theory is a truncation of the  $SO(8) \ltimes \mathbf{T}^{28}$  gauged  $N = 16$  theory with scalar manifold  $E_{8(8)}/SO(16)$  and can be obtained by an  $S^7$  compactification of type I theory in ten dimensions. Domain wall solutions of both gauged supergravities are analytically found and can be uplifted to higher dimensions. These provide domain wall vacua in the three dimensional gauged supergravity framework which might be useful for the study of Domain Wall<sub>3</sub>/QFT<sub>2</sub> correspondence.

**KEYWORDS:** Gauge/Gravity Correspondence and Supergravity Models.

## 1. Introduction

Since the original proposal of the AdS/CFT correspondence [1], there have been a lot of works generalizing this conjecture to a more general case than the correspondence between string theory, or its effective supergravity theory, on an AdS space and a conformal field theory living on the boundary of the AdS space. One of the generalizations is the Domain Wall/Quantum Field Theory (DW/QFT) correspondence [2]. Unlike the maximally supersymmetric AdS spaces, the domain walls are half supersymmetric and have only Poincare symmetry in one dimension lower rather than the full conformal symmetry in the AdS cases. The solutions are supposed to be dual to the QFT in one dimension lower. Similar to the standard AdS/CFT correspondence, the correlation functions of the dual QFT have been studied for example in [3] and [4]. The correspondence also has an application in cosmology in the form of domain wall/cosmology correspondence [5], [6], [7].

In string/M theory, domain walls arise from the near horizon limit of  $p$ -branes with various dimensions except for D3, M2 and M5 branes for which the dilaton is constant or absent. The resulting backgrounds are products of the  $p+2$  dimensional domain wall and sphere. In the dual frame, the corresponding metrics can be written as a product of AdS spaces and spheres with the exception of the five brane [2]. In supergravities which are considered as effective theories of string/M theory, they correspond to domain wall solutions with codimension one of  $p+2$  dimensional gauged supergravity with the gauge group given by the isometry of the sphere on which ten or eleven dimensional theories are compactified.

Domain walls in gauged supergravities in various dimensions have been considered in many places some of which are [8], [9], [10], [11] and [12]. In this work, we focus on the case of three dimensional gauged supergravity of Chern-Simons type. The resulting domain walls should correspond to some QFT in two dimensions. Since gauged supergravity in three dimensions has been systematically explored only in the last decade, not many studies of the associated domain wall solutions have been given so far. To the best of the author's knowledge, the explicit solutions given here are the first example of the domain wall vacua constructed directly in this framework. We hope this work will at least provide a toy model to the study of DW<sub>3</sub>/QFT<sub>2</sub> which, on the other hand, may also give some insights about the correspondence in higher dimensions.

Chern-Simons gauged supergravity constructed in [13], for an earlier construction see [14] and [15], is the convenient framework to begin with because its structure is much simpler than the Yang-Mills type theory. A special class of domain walls in this gauged supergravity in which the solutions approach AdS spaces in some limits, or equivalently interpolate between two AdS critical points of the scalar potential, have

been studied in previous works, see for example [16], [17], [18] and [19]. The resulting solutions have an interpretation of RG flows describing deformations of a UV CFT to another CFT in the IR. In this paper, we will study the solution of the gauged supergravity whose scalar potential does not admit any critical points on the chosen scalar submanifold. This is the analogue of the similar study in higher dimensions mentioned above.

We study two gauged supergravities namely  $N = 4$  and  $N = 8$  theories with non-semisimple gauge groups. In the  $N = 4$  case, the theory describes  $(1, 0)$  supergravity in six dimensions reduced on an  $SU(2)$  group manifold of  $S^3$  topology studied in [20] and [21]. The corresponding scalar manifold is given by  $SO(4, 3)/SO(4) \times SO(3)$  with the gauge group being  $SO(3) \ltimes (\mathbf{T}^3, \hat{\mathbf{T}}^3)$ . This is equivalent to Yang-Mills type gauged supergravity with gauge group  $SO(3) \sim SU(2)$  and scalar manifold  $GL(3, \mathbb{R})/SO(3)$  coupled to three massive vector fields [22]. The  $N = 8$  theory has  $SO(8, 8)/SO(8) \times SO(8)$  as the scalar manifold in which the  $SO(8) \ltimes \mathbf{T}^{28} \subset SO(8, 8)$  is gauged. The theory is equivalent to  $SO(8)$  gauged supergravity of Yang-Mills type and can be obtained from  $S^7$  reduction of type I string theory [22], [23]. The reduced Yang-Mills type theory does not admit an  $AdS_3$  solution but is expected to have a half supersymmetric domain wall solution which we will explicitly construct in this paper. The KK spectrum of this reduction has been given in [24]. Therefore, the domain wall solutions in both theories can be uplifted to higher dimensions and have higher dimensional interpretations. The solution in the  $N = 4$  theory can be uplifted to a disjoint interior branch of a negative-mass self-dual string [20] while the solution in the  $N = 8$  case should be interpreted as a near horizon limit of D1-brane in type I theory. Furthermore, the theory can be thought of as a truncation of the  $SO(8) \ltimes \mathbf{T}^{28}$  maximal gauged supergravity with scalar manifold  $E_{8(8)}/SO(16)$  arising from  $S^7$  reduction of type IIA or IIB theories in ten dimensions [24], [25].

The paper is organized as follow. We give a short discussion on the structure of three dimensional gauged supergravity in section 2. We refer the reader to [13] for the full discussion. We then construct  $N = 4$  gauged supergravity with scalar manifold  $SO(4, 3)/SO(4) \times SO(3)$  and gauge group  $SO(3) \ltimes (\mathbf{T}^3, \hat{\mathbf{T}}^3)$  in section 3. The associated domain wall solution is also given in this section. In section 4, we move on to the  $N = 8$  theory in which the scalar manifold is  $SO(8, 8)/SO(8) \times SO(8)$ , and the gauge group is given by  $SO(8) \ltimes \mathbf{T}^{28}$ . We also find a domain wall solution in this case. The appendix is devoted to some useful formulae for the  $N = 8$  theory. Apart from giving more details, the aim of this appendix is also to give the more detailed construction for further investigations of the resulting scalar potential which is useful in the study of holographic RG flows. We end the paper with some conclusions and comments in section 5. All computations carried out in this work are achieved by using the computer

program *Mathematica*.

## 2. Gauged supergravity in three dimensions

We now give a brief review of three dimensional gauged supergravity in the  $SO(N)$  R-symmetry covariant formulation of [13]. We refer the reader to [13] for both details and notations. The gauged supergravity theory is described by gaugings of non-linear sigma models coupled to supergravity which is by itself topological. In this work, we always consider the case of symmetric scalar target spaces of the form  $G/H$ . Specifically, we are interested in the  $N = 4$  and  $N = 8$  theories with scalar manifolds  $SO(4, 3)/SO(4) \times SO(3)$  and  $SO(8, 8)/SO(8, 8)$ , respectively.

Couplings of the sigma model to supergravity require  $N - 1$  almost complex structures  $f^P$ ,  $P = 2, \dots, N$  which can be used to find the tensors  $f^{IJ}$  as follow:

$$f^{1P} = -f^{P1} = f^P, \quad f^{PQ} = f^{[P} f^{Q]}. \quad (2.1)$$

They generate the  $SO(N)$  R-symmetry in the spinor representation. As in [13], indices  $I, J = 1, \dots, N$  are R-symmetry indices while  $i, j = 1, \dots, d$ , or in the flat basis  $A, B = 1, \dots, d$ , label coordinates on the target space of dimension  $d$ .

In general, scalar manifolds of the  $N = 4$  theory are product of two quaternionic manifolds which need not be symmetric. The R-symmetry  $SO(4) \sim SO(3) \times SO(3)$  acts separately on the two subspaces. We will study the case in which the target space consists of only one quaternionic manifold, sometime called the degenerate case. On the other hand, for the  $N = 8$  theory, supersymmetry requires that the scalar manifolds are of the form  $SO(8, k)/SO(8) \times SO(k)$  [26].  $k$  labels the number of matter supermultiplets. Before moving on, we note some useful formulae for the coset spaces. The coset space  $G/H$  can be described by the coset representative  $L$  transforming by left and right multiplications under the global  $G$  and local  $H$  symmetries. With  $t^{\mathcal{M}}$ ,  $\mathcal{M} = 1, \dots, \dim G$ , denoting  $G$  generators which can be decomposed into  $H = SO(N) \times H'$  generators and the non-compact ones as  $\{T^{IJ}, X^\alpha, Y^A\}$ , for  $\alpha = 1, \dots, \dim H'$  and  $A = 1, \dots, d(\dim(G/H))$ , we have formulae

$$L^{-1} t^{\mathcal{M}} L = \frac{1}{2} \mathcal{V}^{\mathcal{M}}_{IJ} T^{IJ} + \mathcal{V}^{\mathcal{M}}_{\alpha} X^{\alpha} + \mathcal{V}^{\mathcal{M}}_A Y^A, \quad (2.2)$$

$$L^{-1} \partial_i L = \frac{1}{2} Q_i^{IJ} T^{IJ} + Q_i^{\alpha} X^{\alpha} + e_i^A Y^A. \quad (2.3)$$

Generators  $t^{\mathcal{M}}$ 's satisfy the algebra

$$\begin{aligned}
[T^{IJ}, T^{KL}] &= -4\delta^{[I[K}T^{L]J]}, & [T^{IJ}, Y^A] &= -\frac{1}{2}f^{IJ,AB}Y_B, \\
[X^\alpha, X^\beta] &= f^{\alpha\beta}_\gamma X^\gamma, & [X^\alpha, Y^A] &= h^\alpha_B{}^A Y^B, \\
[Y^A, Y^B] &= \frac{1}{4}f_{IJ}^{AB}T^{IJ} + \frac{1}{8}C_{\alpha\beta}h^{\beta AB}X^\alpha
\end{aligned} \tag{2.4}$$

which will be important later on. In the above algebra, the anti-symmetric tensors  $h^\alpha_B{}^A$  generate the  $H'$  algebra in the spinor representation of  $SO(N)$  with structure constants  $f^{\alpha\beta}_\gamma$ , and  $C_{\alpha\beta}$  are symmetric  $H'$  invariant tensors, see [26] for more information about this algebra. The non-compact generators  $Y^A$  transform as a spinor representation of  $SO(N)$ . The tensors  $f^{IJ}$  can be written in terms of  $SO(N)$  gamma matrices  $\Gamma^I$  as

$$f^{IJ} = -\frac{1}{2}\Gamma^{IJ} = -\frac{1}{4}(\Gamma^I\Gamma^J - \Gamma^J\Gamma^I). \tag{2.5}$$

Gaugings can be accomplished by introducing the so-called embedding tensor. In three dimensions, this tensor lives in a symmetric product of two adjoint representations of  $G$ . The product is in turn decomposed into irreducible representations of  $G$ . For viable gaugings, the corresponding embedding tensors have to satisfy two constraints

$$\Theta_{\mathcal{PL}}f^{\mathcal{KL}}{}_{(\mathcal{M}}\Theta_{\mathcal{N})\mathcal{K}} = 0, \tag{2.6}$$

$$\mathbb{P}_{R_0}\Theta_{\mathcal{MN}} = 0. \tag{2.7}$$

The first constraint is a requirement for the gauge generators,  $J_{\mathcal{M}} = \Theta_{\mathcal{MN}}t^{\mathcal{N}}$ , to form a proper subalgebra of  $G$ . The second one comes from supersymmetry. In general, supersymmetry requires the T-tensor, defined below, to satisfy

$$\mathbb{P}_{\boxplus}T^{IJ,KL} = 0 \tag{2.8}$$

where  $\boxplus$  denotes the Riemann tensor-like representation of  $SO(N)$ . But, for symmetric target spaces, this constraint can be uplifted to be the constraint on  $\Theta_{\mathcal{MN}}$  given in (2.7) [13]. The  $R_0$  is a unique  $G$  representation whose branching under  $SO(N)$  contains the  $\boxplus$  representation of  $SO(N)$ . In this case, the condition (2.8) implies (2.7).

Gaugings introduce minimal couplings through covariant derivatives, and the gauge fields enter the gauged Lagrangian via the Chern-Simons terms. To restore supersymmetry broken by the gaugings, fermionic mass-like terms and the scalar potential are needed. These terms are written in terms of the T-tensor given by

$$T_{AB} = \mathcal{V}^{\mathcal{M}}{}_A\Theta_{\mathcal{MN}}\mathcal{V}^{\mathcal{N}}{}_B. \tag{2.9}$$

$\mathcal{A}, \mathcal{B}$  indices decompose into  $\{IJ, \alpha, A\}$  in which  $IJ$  and  $\alpha$  label adjoint representations of  $SO(N)$  and  $H'$ , and  $A$  is the  $SO(N)$  spinor index. The map  $\mathcal{V}^{\mathcal{M}}_{\mathcal{A}}$  can be obtained from (2.2). The  $A_1$  and  $A_2$  tensors are needed in order to compute the scalar potential as well as the supersymmetry transformations of fermions. They are given by

$$\begin{aligned} A_1^{IJ} &= -\frac{4}{N-2}T^{IM,JM} + \frac{2}{(N-1)(N-2)}\delta^{IJ}T^{MN,MN}, \\ A_{2j}^{IJ} &= \frac{2}{N}T^{IJ}_j + \frac{4}{N(N-2)}f_j^{M(I}T^{J)M}_m + \frac{2}{N(N-1)(N-2)}\delta^{IJ}f_j^{KL}T^{KL}_m, \\ V &= -\frac{4}{N}g^2\left(A_1^{IJ}A_1^{IJ} - \frac{1}{2}Ng^{ij}A_{2i}^{IJ}A_{2j}^{IJ}\right). \end{aligned} \quad (2.10)$$

We will also need the supersymmetry transformations of fermions in order to find supersymmetric solutions. There are  $N$  gravitini  $\psi_\mu^I$  and  $d$  spin  $\frac{1}{2}$  fields  $\chi^{iI}$ . Notice that  $\chi^{iI}$  are written in an overcomplete basis. They are subject to the projection constraint given in [13] such that the total number of independent  $\chi^{iI}$  is  $d$ . Indices  $\mu, \nu = 0, 1, 2$  denote three dimensional spacetime coordinates. The supersymmetry transformations are then given by [13]

$$\begin{aligned} \delta\psi_\mu^I &= \mathcal{D}_\mu\epsilon^I + gA_1^{IJ}\gamma_\mu\epsilon^J, \\ \delta\chi^{iI} &= \frac{1}{2}(\delta^{IJ}\mathbf{1} - f^{IJ})^i_j \mathcal{D}\phi^j\epsilon^J - gNA_2^{JIi}\epsilon^J \end{aligned} \quad (2.11)$$

where we have set all the fermionic fields to zero on the right hand side. The covariant derivative  $\mathcal{D}_\mu$  includes the usual spin connection. In this paper, we will use the metric signature  $(-++)$  rather than the Pauli-Kallen metric used in [13], see also the transformation to  $(-++)$  metric given in [26].

We now discuss the relevant form of the embedding tensor used throughout this paper. The gauge group is a non-semisimple group of the form  $G_0 \ltimes (\mathbf{T}^{\dim G_0}, \hat{\mathbf{T}}^M) \subset G$ .  $\mathbf{T}^{\dim G_0}$  consists of  $\dim G_0$  commuting generators transforming in the adjoint representation of  $G_0$  while  $M$  nilpotent generators of  $\hat{\mathbf{T}}^M$  transform in some  $M$  dimensional representation of  $G_0$  and close onto the translational generators  $\mathbf{T}^{\dim G_0}$ . It has been shown in [22] that the corresponding embedding tensor is given by

$$\Theta = g_1\Theta_{\mathcal{A}\mathcal{B}} + g_2\Theta_{\mathcal{B}\mathcal{B}} + g_3\Theta_{\mathcal{H}\mathcal{H}} \quad (2.12)$$

where  $\mathcal{A}, \mathcal{B}$  and  $\mathcal{H}$  refer to  $G_0$ ,  $\mathbf{T}^{\dim G_0}$  and  $\hat{\mathbf{T}}^M$  parts of the full gauge group, respectively. Although the indices  $\mathcal{A}$  and  $\mathcal{B}$  have been used previously as indices of the T-tensors, there should be no confusions here since the latter no longer appear in the rest of the paper. This gauging is on-shell equivalent to Yang-Mills gauged supergravity with gauge group  $G_0$  coupled to  $M$  massive vector fields [22]. Notice that there

is no coupling between the semisimple part with itself. This is a crucial point for the CS-YM equivalence to work [22]. For later convenience, we will also repeat the full gauge algebra from [22]

$$\begin{aligned} [J^m, J^n] &= f^{mn}{}_k J^k, & [J^m, T^n] &= f^{mn}{}_k T^k, & [T^m, T^n] &= 0, \\ [J^m, \hat{T}^\alpha] &= t^{m\alpha}{}_\beta \hat{T}^\beta, & [\hat{T}^\alpha, \hat{T}^\beta] &= t^{\alpha\beta}{}_m T^m, & [T^m, \hat{T}^\alpha] &= 0, \end{aligned} \quad (2.13)$$

where  $J^m$ ,  $T^m$  and  $\hat{T}^\alpha$  are generators of  $G_0$ ,  $\mathbf{T}^{\dim G_0}$  and  $\hat{\mathbf{T}}^M$ , respectively. The  $\Theta_{AB}$  and  $\Theta_{BB}$  are given by the Cartan-Killing form,  $\eta_{mn}$ ,  $m, n = 1, \dots, \dim G_0$ , while the  $\Theta_{\mathcal{HH}}$  is described by  $\kappa_{\alpha\beta}$  given by  $\eta_{mn}$  and the structure constants in (2.13) via the relation  $\eta_{mn} t^{n\alpha}{}_\beta = \kappa_{\beta\gamma} t^{\alpha\gamma}{}_m$  [22]. The couplings  $g_1$ ,  $g_2$  and  $g_3$  are in general not independent. As we will see, consistency conditions will impose some relations between them.

### 3. A domain wall solution in $N = 4$ theory

In this section, we study a domain wall solution in the  $N = 4$  gauged supergravity with  $SO(4, 3)/SO(4) \times SO(3)$  scalar manifold. The gauge group considered here is the non-semisimple group  $SO(3) \ltimes (\mathbf{T}^3, \hat{\mathbf{T}}^3)$ . As mentioned before, this theory describes the reduction of  $N = (1, 0)$  supergravity in six dimensions on an  $SU(2)$  group manifold. The reduced theory is equivalent to the  $SO(3) \ltimes (\mathbf{T}^3, \hat{\mathbf{T}}^3)$  CS type gauged supergravity in three dimensions.

We begin with the structure of the coset manifold. We use the basis elements of the general  $7 \times 7$  matrices

$$(e_{ab})_{cd} = \delta_{ac}\delta_{bd}, \quad a, b = 1, \dots, 7. \quad (3.1)$$

These are generators of  $GL(7, \mathbb{R})$ . There are 12 scalars in the coset  $SO(4, 3)/SO(4) \times SO(3)$ . Under the maximal compact subgroup  $SO(4) \times SO(3) \sim SO(3) \times SO(3) \times SO(3)$ , they transform as

$$(\mathbf{4}, \mathbf{3}) = (\mathbf{2}, \mathbf{2}, \mathbf{3}). \quad (3.2)$$

We can see that the scalars transform as a spinor representation under the first two  $SO(3)$ 's in the  $SO(4)$ . We then identify one of them as the R-symmetry group  $SO(3)_R$ . This choice is certainly not unique. The crucial point in this formulation is the fact that the non-compact generators transform as a spinor of  $SO(3)$ . We can also choose the last  $SO(3)$  to be the R-symmetry provided that we change the basis in such a way that the non-compact generators transform as a spinor representation under this  $SO(3)$ .

To label the three different  $SO(3)$ 's, we use the following identification:  $SO(3)_R \times SO(3)' \subset SO(4)$  and  $SO(3)^{(2)}$  for the  $SO(3)$  in the maximal compact subgroup of

$SO(4, 3)$ . We now construct generators of  $SO(4, 3)$ . The  $SO(4) \times SO(3)$  generators are given by

$$\begin{aligned} SO(3)^{(2)} : \quad J_1^{ij} &= e_{ji} - e_{ij}, \quad i, j = 1, 2, 3, \\ SO(4) : \quad j^{ab} &= e_{b+3, a+3} - e_{a+3, b+3}, \quad a, b = 1, \dots, 4. \end{aligned} \quad (3.3)$$

The corresponding generators of  $SO(3)_R \times SO(3)'$  are then given by

$$\begin{aligned} SO(3)_R : \quad \bar{T}^1 &= j^{12} + j^{34}, \quad \bar{T}^2 = j^{13} - j^{24}, \quad \bar{T}^3 = j^{23} + j^{14}, \\ SO(3)' : \quad \tilde{T} &= j^{12} - j^{34}, \quad \tilde{T}^2 = j^{13} + j^{24}, \quad \tilde{T}^3 = j^{23} - j^{14}. \end{aligned} \quad (3.4)$$

However, it is more convenient in the computation to label  $SO(3)_R$  generators by  $T^{IJ}$  with  $I, J = 1, 2, 3, 4$ . Accordingly, we define the following generators

$$SO(3)_R : \quad T^{IJ} = \frac{1}{2} \left( j^{IJ} + \frac{1}{2} \epsilon_{IJKL} j^{KL} \right), \quad I, J = 1, \dots, 4. \quad (3.5)$$

Finally, the non-compact generators are given by

$$Y^A = \begin{cases} \frac{1}{\sqrt{2}}(e_{1, A+3} + e_{A+3, 1}), & A = 1, \dots, 4, \\ \frac{1}{\sqrt{2}}(e_{2, A-1} + e_{A-1, 2}), & A = 5, \dots, 8, \\ \frac{1}{\sqrt{2}}(e_{3, A-5} + e_{A-5, 3}), & A = 9, \dots, 12. \end{cases} \quad (3.6)$$

We have labeled the non-compact generators such that they fit into our general formulation. Furthermore, the normalization of  $T^{IJ}$  and  $Y^A$  is chosen to satisfy the G-algebra (2.4).

We now move to the generators of the gauge group. First of all, the semisimple part of the gauge group  $SO(3)$  is given by the diagonal of all three  $SO(3)$ 's,  $SO(3) = [SO(3)_R \times SO(3)' \times SO(3)^{(2)}]_{\text{diag}}$ . The corresponding generators are given by

$$J^{ij} = J_1^{ij} + j^{i+1, j+1}, \quad i, j = 1, 2, 3. \quad (3.7)$$

The translational generators are given by

$$t^{ij} = J_1^{ij} - j^{ij} - (e_{j+4, i} + e_{i, j+4}) + (e_{j, i+4} + e_{i+4, j}), \quad i, j = 1, 2, 3. \quad (3.8)$$

It can be verified that they transform as an adjoint representation of  $SO(3)$  and that they commute with each other. Finally, the nilpotent generators of  $\hat{\mathbf{T}}^3$  are found to be

$$\hat{t}^\alpha = e_{\alpha 4} + e_{4\alpha} + e_{4, \alpha+4} - e_{\alpha+4, 4}, \quad \alpha = 1, 2, 3, \quad (3.9)$$



which transform as a vector representation of  $SO(3)$  and close onto  $\mathbf{T}^3$ .

Recall that the general form of the embedding tensor for this gauge group is given by

$$\Theta = g_1 \Theta_{\mathcal{AB}} + g_2 \Theta_{\mathcal{BB}} + g_3 \Theta_{\mathcal{HH}}. \quad (3.10)$$

The symbols  $\mathcal{A}$ ,  $\mathcal{B}$  and  $\mathcal{H}$  denote the  $SO(3)$ ,  $\mathbf{T}^3$  and  $\hat{\mathbf{T}}^3$  parts of the full gauge group, respectively. It turns out that consistency conditions (2.6) and (2.7) require

$$g_2 = 0, \quad g_3 = -g_1 = -g. \quad (3.11)$$

This implies that the  $SO(3) \times \mathbf{T}^3$  is by itself not a consistent gauge group. If this gauge group was possible, the corresponding Yang-Mills type would describe the  $SU(2)$  reduction of the (1, 0) six dimensional theory without any massive vector fields. This is consistent with the fact that the massive vector fields cannot be truncated out without truncating the gauge fields or setting the two-form flux along  $S^3$  to zero. This is because, from the reduced equation of motion for the massive vector fields given in [20], the gauge fields act as a source term for the massive vector fields.

The tensor  $f^{IJ}$  can be computed from the G-algebra (2.4) by

$$f_{AB}^{IJ} = -2\text{Tr}(Y_B [T^{IJ}, Y_A]). \quad (3.12)$$

All the  $\mathcal{V}$  maps are calculated by the following relations coming from (2.2)

$$\begin{aligned} \mathcal{V}_{\mathcal{A}}^{m,IJ} &= -\text{Tr}(L^{-1} X^m L T^{IJ}), & \mathcal{V}_{\mathcal{B}}^{m,IJ} &= -\text{Tr}(L^{-1} S^m L T^{IJ}), & m = 1, 2, 3, \\ \mathcal{V}_{\mathcal{H}}^{m,IJ} &= -\text{Tr}(L^{-1} \hat{t}^m L T^{IJ}), & \mathcal{V}_{\mathcal{A}}^{m,A} &= \text{Tr}(L^{-1} X^m L Y^A), \\ \mathcal{V}_{\mathcal{B}}^{m,A} &= \text{Tr}(L^{-1} S^m L Y^A), & \mathcal{V}_{\mathcal{H}}^{m,A} &= \text{Tr}(L^{-1} \hat{t}^m L Y^A) \end{aligned} \quad (3.13)$$

where for conveniences, we have redefined the gauge generators to be

$$\begin{aligned} X^1 &= J^{12}, & X^2 &= J^{13}, & X^3 &= J^{23}, \\ S^1 &= t^{12}, & S^2 &= t^{13}, & S^3 &= t^{23}. \end{aligned} \quad (3.14)$$

Under the  $SO(3) = [SO(3)_R \times SO(3)' \times SO(3)^{(2)}]_{\text{diag}}$  of the gauge group, the 12 scalars transform as

$$(\mathbf{2} \times \mathbf{2}) \times \mathbf{3} = \mathbf{1} + \mathbf{3}_V + \mathbf{3}_A + \mathbf{5} \quad (3.15)$$

where we have distinguished the two representation  $\mathbf{3}$ 's by the subscripts  $V$  (vector) and  $A$  (anti-symmetric tensor). This decomposition gives a new basis for the 12 scalars.

The above representations correspond to the following non-compact generators

$$\begin{aligned}
\mathbf{1} : \quad & Y^{(1)} = \frac{1}{\sqrt{3}} (Y_2 + Y_7 + Y_{12}), \\
\mathbf{3}_A : \quad & Y_1^{(3)A} = \frac{1}{\sqrt{2}} (Y_3 - Y_6), \quad Y_2^{(3)A} = \frac{1}{\sqrt{2}} (Y_4 - Y_{10}), \\
& Y_3^{(3)A} = \frac{1}{\sqrt{2}} (Y_8 - Y_{11}), \\
\mathbf{3}_V : \quad & Y_1^{(3)V} = Y_1, \quad Y_2^{(3)V} = Y_5, \quad Y_3^{(3)V} = Y_9, \\
\mathbf{5} : \quad & Y_1^{(5)} = \frac{1}{\sqrt{2}} (Y_3 + Y_6), \quad Y_2^{(5)} = \frac{1}{\sqrt{2}} (Y_4 + Y_{10}), \\
& Y_3^{(5)} = \frac{1}{\sqrt{2}} (Y_8 + Y_{11}), \quad Y_4^{(5)} = \frac{1}{\sqrt{2}} (Y_2 - Y_7), \\
& Y_5^{(5)} = \frac{1}{\sqrt{3}} (Y_2 + Y_7 - 2Y_{12}). \tag{3.16}
\end{aligned}$$

Given the form of the coset representative  $L$ , we can now compute the T-tensor,  $A_1$  and  $A_2$  tensors and finally the scalar potential from the embedding tensor via the formulae give in the previous section. Explicitly, the relevant components of the T-tensor are given by

$$\begin{aligned}
T^{IJ,KL} = & g(\mathcal{V}_A^{m,IJ} \mathcal{V}_B^{m,KL} + \mathcal{V}_B^{m,IJ} \mathcal{V}_A^{m,KL}) \\
& - g \mathcal{V}_H^{m,IJ} \mathcal{V}_H^{m,KL} \tag{3.17}
\end{aligned}$$

$$\begin{aligned}
T^{IJ,A} = & g(\mathcal{V}_A^{m,IJ} \mathcal{V}_B^{m,A} + \mathcal{V}_B^{m,IJ} \mathcal{V}_A^{m,A}) \\
& - g \mathcal{V}_H^{m,IJ} \mathcal{V}_H^{m,A}. \tag{3.18}
\end{aligned}$$

It turns out to be very complicated to compute the scalar potential for all 12 scalars simultaneously. We then study the potential in each sector according to the representations under  $SO(3)$  given above. For scalar in the  $\mathbf{5}$ ,  $\mathbf{3}_A$  and  $\mathbf{3}_V$ , there are no interesting non-trivial critical points. Therefore, we will not study these cases further. Moreover, we will not give the explicit form of the resulting potential since they are not relevant for this work.

When turning on the  $SO(3)$  singlet scalar, we find that the scalar potential does not have any critical points. Rather than a maximally supersymmetric  $AdS_3$  critical point at  $L = \mathbf{I}$ , there is a domain wall solution preserving half of the supersymmetry as we will see. We begin with the coset representative

$$L = e^{a(r)Y^{(1)}}. \tag{3.19}$$

The scalar potential is computed to be

$$V = -96e^{\sqrt{\frac{8}{3}}a(r)}g^2 \tag{3.20}$$

which clearly does not have any critical points. We then expect to find a domain wall solution in this case. Notice that the potential take the same form as those studied in higher dimensional gauged supergravities.

We want to find a half-supersymmetric solution, so we begin with the BPS equations coming from setting the supersymmetry variations of the fermionic fields  $\psi_\mu^I$  and  $\chi^{iI}$  to zero. The metric is given by the usual domain wall ansatz

$$ds^2 = e^{2A(r)} dx_{1,1}^2 + dr^2. \quad (3.21)$$

Generally, in the  $D$  dimensional domain wall ansatz, we want the  $D - 1$  dimensional Poincare symmetry to be preserved. Therefore, the function  $A$  can depend only on the radial coordinate  $r$ . Any function in front of  $dr^2$  can be absorbed by the redefinition of  $r$ . So, the above ansatz is the general ansatz for the domain wall preserving two dimensional Poincare symmetry  $ISO(1, 1)$ . The ansatz is similar to that in the study of holographic RG flows in which the metric interpolates between two  $AdS_3$  spaces. In that case, the function  $A(r)$  behaves linearly in  $r$  at both ends. It is well-known that an AdS space is a special case of domain walls for which the isometry gets enhanced from  $ISO(1, 1)$  to  $SO(2, 2)$  which is the isometry of  $AdS_3$ . The number of supersymmetry is twice that of the domain wall as well.

The solutions studied here are domain walls that do not give  $AdS_3$  as a limiting case. We will explicitly see below that the solution for  $A(r)$  will not be linear in any limits of  $r$ . The solution will rather be a flat space  $\mathbb{R}^{1,2}$  at one limit in  $r$ , at  $r = 0$  or  $r = \infty$ . We now put the corresponding spin connection computed from the metric into the BPS equations  $\delta\chi^{iI} = 0$  and  $\delta\psi_\mu^I = 0$  for  $\mu = 0, 1$ . With the condition  $\gamma_r \epsilon^I = \epsilon^I$ , the former gives

$$a' + 2\sqrt{6}e^{\sqrt{\frac{2}{3}}a}g = 0 \quad (3.22)$$

while the latter gives

$$A' - 12e^{\sqrt{\frac{2}{3}}a}g = 0. \quad (3.23)$$

Due to the projection by  $\gamma_r$ , the resulting solution will be half-supersymmetric. In the above equations and the remaining ones in the paper, we have used the notation  $a' = \frac{da}{dr}$ . Equation (3.22) can be easily solved to obtain

$$a(r) = -\sqrt{\frac{3}{2}} \ln \left( 8gr - \sqrt{\frac{2}{3}}C_1 \right) \quad (3.24)$$

where  $C_1$  is a constant. Inserting the solution for  $a$  into (3.23), we can solve for the  $A$  solution

$$A(r) = C_2 + \frac{3}{2} \ln \left( 24gr - \sqrt{6}C_1 \right). \quad (3.25)$$

The  $\delta\psi_r^I = 0$  equation gives the condition on the  $r$ -dependent Killing spinors. With the ansatz  $\epsilon^I = e^{f(r)}\epsilon_0^I$  for  $\gamma_r\epsilon_0^I = \epsilon_0^I$ , it can be easily verified that  $\delta\psi_r^I = 0$  equation is satisfied by

$$\epsilon^I = e^{\frac{A}{2}}\epsilon_0^I \quad (3.26)$$

similar to the corresponding solutions in higher dimensions.

The constant  $C_2$  can be set to zero by rescaling the coordinates  $x^0$  and  $x^1$ . On the other hand, by shifting the coordinate  $r$ , we can remove the constant  $C_1$ . The metric is then given by

$$ds^2 = (24g_1r)^3 dx_{1,1}^2 + dr^2. \quad (3.27)$$

We can also write it in the form of the warped  $AdS_3$ . To do this, we first rescale all of the coordinates to the dimensionless ones, recalling that the coupling  $g$  has a dimension of mass,

$$\tilde{x}^0 = gx^0, \quad \tilde{x}^1 = gx^1, \quad \text{and} \quad \tilde{r} = gr. \quad (3.28)$$

The metric (3.27) can then be rewritten as

$$ds^2 = \frac{1}{3^{15}(2^{12})g^2}\rho^{-4} \left( \frac{dx_{1,1}^2 + d\rho^2}{\rho^2} \right) \quad (3.29)$$

by using the new coordinate  $\rho = -\frac{2}{3(24)^{\frac{3}{2}}}r^{-\frac{1}{2}}$ .

## 4. A domain wall solution in $N = 8$ theory

In this section, we will study  $N = 8$  gauged supergravity in three dimensions with scalar target space  $SO(8,8)/SO(8) \times SO(8)$ . This theory has also been studied in [17] with another gauge group  $(SO(4) \ltimes \mathbf{T}^6) \times (SO(4) \ltimes \mathbf{T}^6)$ . In that case, the theory is supposed to describe the nine dimensional supergravity reduced on  $S^3 \times S^3$ . Here, we will study the same theory with gauge group  $SO(8) \ltimes \mathbf{T}^{28}$ . According to the equivalence to the  $SO(8)$  Yang-Mills gauged supergravity [22], this theory should describe the compactification of the ten dimensional type I theory on  $S^7$ .

As in the  $N = 4$  theory, we still use the general formulation of [13]. We begin with the structure of  $SO(8,8)/SO(8) \times SO(8)$  coset. We will use the coset representative in the fundamental representation  $\mathbf{16}$  of  $SO(8,8)$ . The 64 scalars transform as a bivector,  $(\mathbf{8}, \mathbf{8})$ , of the two  $SO(8)$ 's. However, we need the scalars to transform as  $(\mathbf{8}_s, \mathbf{8})$  in order to fit into the  $SO(N)$  covariant formulation of [13]. We note here our notations regarding to the  $SO(8)$  representations. The vector representation of  $SO(8)$  is simply denoted by  $\mathbf{8}$  while the spinor and conjugate spinor representations are denoted by  $\mathbf{8}_s$  and  $\mathbf{8}_c$ , respectively.

Similar to [17], we will use the  $SO(8)$  R-symmetry in the spinor representation of the form

$$T^{IJ} = \begin{pmatrix} \Gamma^{IJ} & 0 \\ 0 & 0 \end{pmatrix} \quad (4.1)$$

where  $\Gamma^{IJ} = -\frac{1}{4}(\Gamma^I(\Gamma^J)^T - \Gamma^J(\Gamma^I)^T)$ . The  $\Gamma^I$  are  $8 \times 8$  gamma matrices of  $SO(8)$  whose explicit forms are given in the appendix. They are embedded in the full Dirac gamma matrices  $\gamma^I$  as

$$\gamma^I = \begin{pmatrix} 0 & \Gamma^I \\ (\Gamma^I)^T & 0 \end{pmatrix}. \quad (4.2)$$

It can be easily verified that  $T^{IJ}$  satisfy  $SO(8)$  algebra given in (2.4). We have used the same notation for  $SO(8)$  Dirac gamma matrices and the spacetimes gamma matrices, but this should not be confusing since the former will not appear elsewhere in the paper.

In order to construct the  $SO(8,8)/SO(8) \times SO(8)$  coset, we define the generators of  $GL(16, \mathbb{R})$

$$(e_{mn})_{pq} = \delta_{mp}\delta_{nq}, \quad m, n, p, q = 1, \dots, 16. \quad (4.3)$$

The maximal compact subgroup  $SO(8)^{(1)} \times SO(8)^{(2)}$  is generated by

$$\begin{aligned} SO(8)^{(1)} : \quad & J_1^{ab} = e_{ba} - e_{ab}, \quad a, b = 1, \dots, 8, \\ SO(8)^{(2)} : \quad & J_2^{rs} = e_{s+8, r+8} - e_{r+8, s+8}, \quad r, s = 1, \dots, 8. \end{aligned} \quad (4.4)$$

The non-compact generators transforming in the  $(\mathbf{8}, \mathbf{8})$  representation of  $SO(8)^{(1)} \times SO(8)^{(2)}$  are given by

$$Y^{ar} = e_{a, r+8} + e_{r+8, a}, \quad r, a = 1, \dots, 8. \quad (4.5)$$

We now come to  $SO(8) \ltimes \mathbf{T}^{28}$  generators. The semisimple part is given by the diagonal subgroup of  $SO(8)^{(1)} \times SO(8)^{(2)}$ . Including the 28 commuting generators,  $t^{ab} = t^{[ab]}$ , of  $\mathbf{T}^{28}$ , the full gauge generators are given by

$$\begin{aligned} SO(8) : \quad & J^{ab} = J_1^{ab} + J_2^{ab}, \\ \mathbf{T}^{28} : \quad & t^{ab} = J_1^{ab} - J_2^{ab} + Y^{ba} - Y^{ab}. \end{aligned} \quad (4.6)$$

We can check that they satisfy the algebra (2.13). Similar to [17], the  $f^{IJ}$  tensor is defined by

$$f_{ar, bs}^{IJ} = -\text{Tr}(Y_{bs} [T^{IJ}, Y_{ar}]). \quad (4.7)$$

We then consider the embedding tensor. The general form of the embedding tensor consists of the couplings between  $\mathcal{AB}$  and  $\mathcal{BB}$  parts. Consistency conditions require

the missing of the  $\mathcal{BB}$  part. Therefore, the final form of the embedding tensor takes the form

$$\Theta = g\Theta_{\mathcal{AB}}. \quad (4.8)$$

This form is similar to the embedding tensor of the same gauge group in the  $N = 16$  theory studied in [25]. This is not unexpected since the  $N = 8$  theory considered here can be obtained from a truncation of the  $N = 16$  theory. With the T-sensor given in the appendix, we can compute the scalar potential and set up the BPS equations from supersymmetry transformations of fermions as in the  $N = 4$  theory of the previous section.

The full scalar manifold is 64 dimensional, but, in this work, we are interested in the domain wall solution which involves only the dilaton. This scalar is invariant under the  $SO(8)$  part of the gauge group. Under  $SO(8) \subset (SO(8)^{(1)} \times SO(8)^{(2)})_{\text{diag}}$ , the 64 scalars transform as

$$\mathbf{8} \times \mathbf{8} = \mathbf{1} + \mathbf{28} + \mathbf{35}. \quad (4.9)$$

The singlet corresponds to the dilation we are considering and corresponds to the non-compact generator

$$Y_s = Y_{11} + Y_{22} + Y_{33} + Y_{44} + Y_{55} + Y_{66} + Y_{77} + Y_{88}. \quad (4.10)$$

The coset representative is parametrized by

$$L = e^{\phi(r)Y_s}. \quad (4.11)$$

As expected, the potential turns out to be

$$V = -192g^2e^{4\phi} \quad (4.12)$$

which again does not admit any critical points.

We then find the associated domain wall solution. The metric ansatz is again given by (3.21). Together with the projection condition  $\gamma_r\epsilon^I = \epsilon^I$ , the BPS equation coming from  $\delta\chi^{iI} = 0$  gives

$$\phi' - 2ge^{2\phi} = 0. \quad (4.13)$$

The solution is readily obtained

$$\phi = -\frac{1}{2}\ln(C_3 - 4gr). \quad (4.14)$$

Equation  $\delta\psi_\mu^I = 0$  reads

$$A' + 16ge^{2\phi} = 0. \quad (4.15)$$

After substituting  $\phi$  from (4.14), we find

$$A = 4 \ln (C_3 - 4gr). \quad (4.16)$$

From this, the metric takes the form

$$ds^2 = (2gr)^8 dx_{1,1}^2 + dr^2. \quad (4.17)$$

As in the  $N = 4$  case, the 8 Killing spinors are given by  $\epsilon^I = e^{\frac{A}{2}} \epsilon_0^I$  in which  $\gamma_r \epsilon_0^I = \epsilon_0^I$ .

Similar to the  $N = 4$  case, after rescaling the coordinates  $x^0, x^1$  and  $r$  to  $x^0 g, x^1 g$  and  $rg$  as well as changing to the new coordinate  $\rho$ , we end up with the metric in the warped  $AdS_3$  form

$$ds^2 = \rho^{-\frac{2}{3}} \ell^2 \left( \frac{dx_{1,1}^2 + d\rho^2}{\rho^2} \right) \quad (4.18)$$

where

$$\rho = -\frac{1}{2^6 r^3} \quad \text{and} \quad \ell^2 = \frac{1}{2^{16} g^2}. \quad (4.19)$$

It is also interesting to study the scalar potential in more details. We can use the truncation introduced by [27] from which the consistency of our truncation to the  $SO(8)$  singlet follows. For example, we can consider residual symmetries of the form  $SO(4)_{\text{diag}}, SO(4)_{\text{diag}} \times SO(4)_{\text{diag}}$  or  $SO(3)_{\text{diag}}$ . Among the 64 scalars, there are four singlets under the  $SO(4)_{\text{diag}} \subset (SO(4)^{(1)+} \times SO(4)^{(1)-} \times SO(4)^{(2)+} \times SO(4)^{(2)-})_{\text{diag}}$ . The  $SO(4)^{(1)\pm, (2)\pm}$  are subgroups of  $SO(8)^{(1), (2)}$ . If we consider only  $SO(3) \subset SO(4)$  in which  $\mathbf{4} \rightarrow \mathbf{3} + \mathbf{1}$ , there are eight singlets under  $SO(3)_{\text{diag}} \subset (SO(3)^{(1)+} \times SO(3)^{(1)-} \times SO(3)^{(2)+} \times SO(3)^{(2)-})_{\text{diag}}$ . Under  $SO(4)^{(1)+} \times SO(4)^{(2)+}$ , there are 16 singlets. It turns out that even with only 4 singlets, the potential computation is very complicated and takes a very long time. So, we postpone this analysis to future works.

## 5. Conclusions

In this paper, we have studied two domain wall solutions in Chern-Simons three dimensional gauged supergravity. The solutions are half supersymmetric and should correspond to dual QFT's in two dimensions with  $(4, 0)$  and  $(8, 0)$  supersymmetries, respectively.

In the  $N = 4$  theory, the solution can be uplifted to six dimensions via the  $SU(2)$  reduction ansatz given in [20]. And, the resulting solution in six dimensions can be thought of as a solution of the ten dimensional heterotic string theory compactified on  $K3$  [28]. Furthermore, the six dimensional solution is interpreted as a negative mass self-dual string according to the discussion in [20], see also [29]. According to the analysis on the scalar submanifolds of  $\mathbf{5}, \mathbf{3}_A, \mathbf{3}_V$  scalars, the scalar potential, most probably,

may not have any critical points at all. Therefore, the theory may not be very useful in the study of holographic RG flows. However, further investigations on the bigger scalar submanifolds or even on the full scalar manifold are needed.

In the  $N = 8$  theory, after uplifting to ten dimensions, the solution should correspond to the near horizon limit of the D1-brane in type I theory in which the associated isometry is  $ISO(1, 1) \times SO(8)$ . The solution can be uplifted to ten dimensions by using the  $S^7$  reduction ansatz given in [23] after integrating out the 28 translational scalars corresponding to the  $\mathbf{T}^{28}$  generators. The resulting ten dimensional metric can be used to study the dual QFT in a similar way as [24]. Regarding the  $N = 8$  theory considered here as a truncation of the  $SO(8) \ltimes \mathbf{T}^{28}$  maximal gauged  $N = 16$  theory via the embedding of the  $SO(8, 8)/SO(8) \times SO(8)$  coset into  $E_{8(8)}/SO(16)$ , the solution given in this work should have the analogue in the maximal gauged supergravity. The latter would fit into a recent classification of domain walls in maximal gauged supergravities [30]. Furthermore, the solutions found in this work may provide a toy model for the study of DW/Cosmology. Finally, the results of this paper hopefully might give a clarification to the vacuum structure of the two gauged supergravities.

In the holographic RG flow context, it is interesting to investigate the scalar potential of the  $N = 8$  theory in more details to see whether, in the absent of the dilaton, there exist any interesting critical points and possibilities of RG flow solutions interpolating between them. The corresponding solution should describe a deformation of the dual  $(8, 0)$  two dimensional field theory. Although this seems to be less likely at least from the partial analysis by the present author, it would be interesting to have a definite conclusion. Apart from the RG flow solution in the compact gauge group  $SO(4) \times SO(4)$  of [16], no other solutions in the  $N = 8$  theory are known. The  $N = 8$  theory with  $SO(8, 8)/SO(8) \times SO(8)$  scalar manifold and non-semisimple gauge group  $(SO(4) \ltimes \mathbf{T}^6) \times (SO(4) \ltimes \mathbf{T}^6)$  has been studied previously in [17], but in that work, there have not been any possible RG flows. The flow in this case would describe a deformation of the large  $N = (4, 4)$  CFT arising from the near horizon limit of the double D1-D5 system.

## Acknowledgement

This work is partially supported by Thailand Center of Excellence in Physics through the ThEP/CU/2-RE3/11 project and Chulalongkorn University through Ratchadapisek Sompote Endowment Fund.

## A. Useful formulae for $N = 8$ theory

In this appendix, we give some useful formulae relevant for the construction studied in



the main text. The  $SO(8)$  gamma matrices are explicitly given by

$$\begin{aligned}
\Gamma_1 &= \sigma_4 \otimes \sigma_4 \otimes \sigma_4, & \Gamma_2 &= \sigma_1 \otimes \sigma_3 \otimes \sigma_4, \\
\Gamma_3 &= \sigma_4 \otimes \sigma_1 \otimes \sigma_3, & \Gamma_4 &= \sigma_3 \otimes \sigma_4 \otimes \sigma_1, \\
\Gamma_5 &= \sigma_1 \otimes \sigma_2 \otimes \sigma_4, & \Gamma_6 &= \sigma_4 \otimes \sigma_1 \otimes \sigma_2, \\
\Gamma_7 &= \sigma_2 \otimes \sigma_4 \otimes \sigma_1, & \Gamma_8 &= \sigma_1 \otimes \sigma_1 \otimes \sigma_1
\end{aligned} \tag{A.1}$$

where

$$\begin{aligned}
\sigma_1 &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, & \sigma_2 &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \\
\sigma_3 &= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, & \sigma_4 &= \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.
\end{aligned} \tag{A.2}$$

The  $\mathcal{V}$  maps are given by

$$\begin{aligned}
\mathcal{V}_A^{ab,IJ} &= -\frac{1}{2} \text{Tr}(L^{-1} J^{ab} T^{IJ}), & \mathcal{V}_B^{ab,IJ} &= -\frac{1}{2} \text{Tr}(L^{-1} t^{ab} T^{IJ}), \\
\mathcal{V}_A^{ab,cr} &= \frac{1}{2} \text{Tr}(L^{-1} J^{ab} Y^{cr}), & \mathcal{V}_B^{ab,cr} &= \frac{1}{2} \text{Tr}(L^{-1} t^{ab} Y^{cr}).
\end{aligned} \tag{A.3}$$

With the embedding tensor given in section 4, the T-tensor can be computed by

$$\begin{aligned}
T^{IJ,KL} &= g \left( \mathcal{V}_A^{ab,IJ} \mathcal{V}_B^{ab,KL} + \mathcal{V}_B^{ab,IJ} \mathcal{V}_A^{ab,KL} \right), \\
T^{IJ,cr} &= g \left( \mathcal{V}_A^{ab,IJ} \mathcal{V}_B^{ab,cr} + \mathcal{V}_B^{ab,IJ} \mathcal{V}_A^{ab,cr} \right)
\end{aligned} \tag{A.4}$$

where the summation over  $a, b$  indices is understood.

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